Subgoal Identification and Error Analysis in Problem Solving

Kimberly Morton**, Edward W. Thomas*, Neff Walker** and Jack Marr**
School of Physics* and Psychology**, Georgia Institute of Technology,
Atlanta GA 30332

Abstract

Quantitative problem solving is the major ingredient of core pre-engineering physics courses. As such, the analysis of problem solving behaviors in these courses can provide an important early source of information not only about how students approach problems in general, but how their performance might be enhanced through the development and assessment of instructional innovations. Any improvement in general problem-solving skills will have obvious benefits in later engineering courses and in career development. We have focused on the electromagnetics portion of our introductory Physics sequence. In common with calculus-based physics courses across the country, this course has a failure rate (D's and F's) exceeding 30%, despite the above average ability and preparation of the students entering the course. Thus, any system enhancing problem solving in this difficult course will significantly impact many students. A principal barrier to successful problem solving is lack of generalization; that is, the ability to transfer knowledge and skills from one problem to another and to recognize classes of problems. The common tendency is to treat each problem as a separate entity to be approached by rigid application of memorized formulae, or simply trial and error. Seemingly small changes in requirements or conditions can be met with bafflement. We have approached problem solving in two interlocked ways. One is the systematic analysis and classification of errors students make when attempting standard problems on quizzes and exams. The second consists of breaking problems down into a sequence of subgoals. The results show that students often improperly identify both the correct operational equations and the appropriate subgoals required for solution. Our analysis will be used to teach students more effective problem solving strategies with a computerized tutorial.

Introduction

The solution of numerical problems is used in instruction to reinforce learning and to illustrate physical principles. This is especially prevalent in an engineering curriculum. Assessment of knowledge is frequently based entirely on solving problems on tests and quizzes, ultimately determining the student's success or failure. Therefore the development of problem solving skills is an important objective for a pre-engineering physics course. An analysis of the procedures used by engineering students to solve numerical problems was performed, using the final exam in an introductory physics course. The objective was to identify the point in the solution procedure(s) where the student makes errors and the solution becomes incorrect.

We studied student problem solving methods and performance in a one quarter calculus based Electricity and Magnetism course. This course is the second in a sequence of three introductory physics courses, and is a required course for all students seeking engineering degrees. The class size is typically 150 students, so the opportunity for direct interaction between faculty and students is limited. Historically, student performance in this course is poor, with around 30% of students failing to make an acceptable grade for engineering majors. The quiz and final exam questions are all multiple choice, and are similar to those found at the end of chapters in a typical text used at this level. In most cases, the incorrect answers are carefully designed to represent an incorrect manipulation of the information given in the problem. Therefore, unless chosen randomly, most incorrect answers represent a definite identifiable mistake and not a procedural error such as an incorrect calculator keystroke. The analyses discussed in this paper are all related to the performance of the students on their final examination. The final grade awarded for the course included laboratory work, homework problem sets, quizzes and the final examination.

Categorization and Problem Complexity

One source of error associated with novice problem solvers is the initial identification of the problem type or category. Research has shown that novice problem solvers categorize problems based on superficial features, particularly from figures [1]. This initial categorization determines the selection of solution method, therefore an incorrect categorization generally leads to uncorrectable errors. Several of the questions included in the exam which was analyzed have similar superficial features, but ask different questions requiring different solutions. These problems are analyzed to determine whether students are categorizing problems improperly, and what effect this has on the number of correct responses.

Another source of error can be related to problem complexity. Some problems involve a single step for solution, and are inherently simple given use of the correct equation. As an example consider the following.

> "What is the magnitude of the magnetic field 3 meters from a long straight wire carrying a current of 1000 Amps?"

This problem requires the student to recall a single operational equation (based on a fundamental law) and then to directly substitute the two numbers provided. Student success on single step questions is typically very high, up to 95% or more.

A more complex problem would be:

> "Two parallel cables are separated by 2 meters and each is 3 meters above the ground. The cables carry currents of 1000 amps each, but in opposite directions. Which of the following expressions represents the magnetic field at the ground level immediately below one of the cables?"

The steps involved in solving this problem are to calculate separately the magnetic fields generated by each cable, which may be given as vectors with horizontal and vertical components, and to combine the vectors correctly, requiring an understanding of the directions of each. These distinct steps in the solution of the problem are considered subgoals, or intermediate manipulations of the given data which are required in order to solve the problem correctly.

Opportunities for error in the more complex problem include incorrect identification of the required law or operational equation, incorrect identification of the magnetic field directions (some students will take them radial to the cable, analogous to an electrostatic field situation), incorrect resolving of field components, as well as other errors in calculations and geometry. This particular problem draws a 70 - 75% correct response. The physics, and the underlying equation are, however, identical to the single concept problem discussed earlier where success is 95%. Since the problems require the same basic knowledge, the decrease in level of performance is associated with the students' difficulty in identifying and/or performing the required intermediate steps (subgoals).

One reason students experience this difficulty is that novice problem solvers tend to focus on superficial details and remember mathematical procedures for solving problems without understanding what each step contributes to the overall solution. Research has shown that students who learn to solve problems using subgoals are better at solving novel problems. Therefore an understanding of the significance of the intermediate steps, or subgoals, improves transfer of knowledge to novel problems [2].

Method for Classification of Errors

Instructors generally solve problems and teach problem solving using a specific method which consists of several key components. In turn, it is expected that the student will use those same components when solving subsequent problems. These components combined represent a general solution procedure. We propose that a well documented solution to a complex multi-step problem should include the following components.

1. The fundamental law or equation on which the solution is based
2. An operational equation, which addresses the specific question asked in the problem
3. A figure, where appropriate
4. Identification and performance of any required intermediate steps
5. Attention to correct use of calculations, geometry (where required) and units

Although the fundamental equation and a figure are not absolutely required for calculating the correct answer to a problem, they both serve important purposes. The fundamental equation is both an indicator of correct categorization of problem type and a basis for deriving the operational equation appropriate to a given problem. The figure serves to clarify the problem, aid in developing correct equations and properly evaluating problem geometry, and can be an indicator of conceptual understanding of problem components.

Final exam papers and worksheets were analyzed for one section of the Electromagnetism course. The students were not told ahead of time that their work would be evaluated, beyond the grade toward the course. This method of analysis has some inherent limitations, since the multiple choice format does not encourage students to write out complete and organized solutions for each problem. Therefore some information of interest was not available. Additionally, students could select either correct or incorrect answers by guessing. However, the available information does reflect the actual methods remembered and used by students, without the aid of example problems or the textbook. The students were allowed to use a 3x5 card of notes/equations during the exam.

The students' written solutions for each exam question were compared with "ideal" solutions in terms of eight components, which are related to the five solution components listed above: fundamental equation, operational equation, figure, identifying need for subgoals, selecting and using appropriate subgoals, calculations, geometry and units. Each component was classified as being: correct, incorrect/incomplete, missing/cannot be determined, or not applicable. A smaller sample of exams was coded by a second reader, in order to establish the reliability of the classification scheme. The students' final grades in the course were also recorded, so that results could be compared for students who performed well and students who performed poorly.

Results

Detailed classifications were made of the responses of 42 students to 14 separate questions, with the analysis in terms of the eight parameters listed previously. Overall results provide some idea of how much information was obtained from the students' worksheets. Correct and incorrect/incomplete information comprised 53% of the data, the remaining 47% was either missing or not applicable. The amount of information gained was more than expected, given the format of the test and that students were given no additional credit for showing their work. Space does not permit a full display of the results and indeed many of the details must be understood in the context of the individual questions. We shall discuss here only the overall trends. Figure 1 shows the percentage of students in each letter grade category who provided satisfactory information on certain of the analyzed parameters.

First notice as a reference the percentage of correct answers; these decrease from 80% for students who achieved an A to 25% for those who received an F; with the multiple choice format of the test a 25% correct rate is little better than guessing. The data shows that less than 20% of students draw an adequate diagram and less than 50% write down the basic law governing the situation. These frequencies are both less than the rate of correct responding in most grade categories. The students, even when successful in the final answer, are therefore not utilizing these solution components.

The frequency with which students identify (and write down) the correct operational equations to employ for the problem and correctly identify the need for subgoals or subsidiary manipulations roughly parallels the overall rate of correct answering. Thus the student's

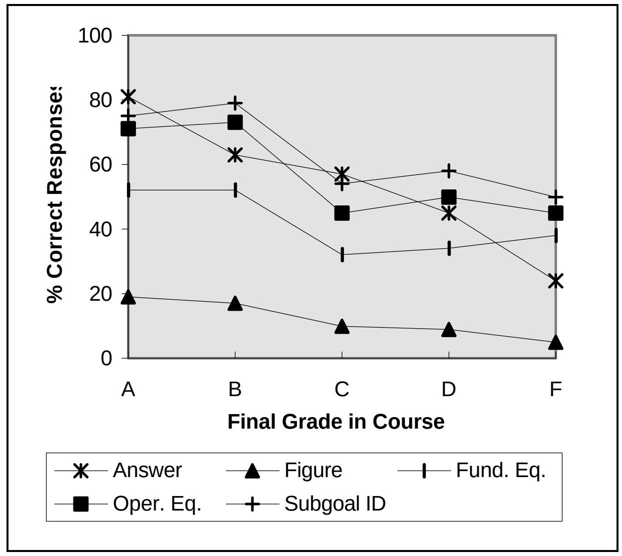

Fig 1. Shown, as a function of the student's ultimate class grade, are the frequency with which certain parameters of the question answer are correctly reported on the student's worksheet. Note first the overall correct responding to the question. Also shown is the percentage of cases where the figure is correctly labeled, the fundamental equation is correctly quoted, the operational equation is correctly stated and the subgoals are correctly identified.

key to successful answering is not analysis, but rather remembering a suitable format. If the operational equation is correctly stated and the subgoals correctly identified, then there are few subsequent errors in mathematics or in data manipulation. There is however a divergence from this trend for those students who attain grades of F. For these the rate of correct identification of equations and the need for subgoals is twice the rate at which they achieve correct answers. In a sense these students may have some idea how the problem should be handled, but the failure lies in the execution.

What about those cases where the operational equations are not correctly written down? In approximately 20% of all cases the operational equation is wrong, this is almost independent of ultimate grade. Thus for A and B students where approximately 20 to 25% do not state the correct operational equation, they are in fact stating an incorrect equation. Rarely do they fail to write down any equation. For the C, D and F students where the operational equation is stated correctly only 50% of the time and stated incorrectly 20% of the time, the remaining 30% wrote no equation at all.

We have designed groups of questions which have high degrees of superficial similarity but differ in the basic physics, and use these to test transfer of knowledge to new scenarios. As an example we have one question to calculate deflection of an electron passing through an electric field and a second question to calculate deflection of an electron through a magnetic field. The electric field problem is specifically addressed in the text and the magnetic field problem is not found specifically in the text. The electric field problem is handled successfully by 67% of the students; the typical answer commences by identifying the acceleration in magnitude and direction followed by application to the dynamical problem. For the deflection in a magnetic field the success rate is only 5%, less than random guessing. The students attempt to handle this case in the same manner as the deflection in the electric field, with the acceleration being taken as constant in magnitude and direction. In the absence of specific experience they adopt as a template a superficially similar problem; they do not analyze the problem to determine whether the solution template is valid.

Conclusions

Successful students move directly to identify the required operational equation and the necessary subgoals. The students appear to be using their previous experience with similar problems as a template, and faults in the data manipulation are rare. Where the students are unsuccessful it is generally through inability to correctly identify the scenario and to revise their solution procedures to accommodate differences between the problem to be solved and their solution template. Students whether successful or not, rarely analyze the problem with a statement of fundamental law or through a detailed diagrammatic representation.

The results suggest that while the teacher may seek to enhance student scores by emphasis on basic understanding, this is not the natural route by which successful students approach a problem. The natural route which the student adopts is to use past experiences and to recall the pragmatic solution to a case learned from previous examples. This is broadly consistent with our previous work [3] where we demonstrated that repetitive exercises in problem solving lead to significant improvement in the rate of correct answering on final examinations.

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