# Discrete Mathematics Assessment Using Learning Objectives Based on Bloom's Taxonomy

Timothy Highley and Anne E. Edlin
La Salle University, highley@lasalle.edu, edlin@lasalle.edu

*Abstract* - **We have developed grading criteria using learning objectives inspired by Bloom's taxonomy for a two-course Discrete Mathematics sequence. For each topic in the courses we developed a hierarchy of learning objectives where the lower-level objectives correspond to the lower levels of Bloom's taxonomy and the higher-level learning objectives require deeper understanding as in the higher levels of Bloom's taxonomy. The grading system was designed to directly link a student's level of comprehension to his or her grade while maintaining the structure of a more traditional course. The grading system clarifies course expectations, helps students to see clearly where they need improvement, and assesses student's total achievement rather than their rate of achievement. Students are evaluated based on their progress with the learning objectives, primarily through quizzes and exams. Because traditional quizzes and exams are used, the assessment method can be implemented without affecting other instructional strategies. We discuss the benefits and challenges of this system along with modifications that help to address some of the challenges we experienced.**

*Index Terms* – Assessment, Bloom's Taxonomy, Discrete Mathematics, Learning Objectives

## INTRODUCTION

The Taxonomy of Educational Objectives often called Bloom's Taxonomy looks at the learning objectives set by educators and classifies them into a hierarchical system based on a student's depth of understanding of the material [1]. We have developed grading criteria using learning objectives inspired by Bloom's Taxonomy for a two-course Discrete Mathematics sequence. The grading system was designed to directly link a student's level of comprehension to his or her grade while maintaining the structure of a more traditional course.

Ideally, the grades students receive correspond to their level of understanding. The deeper a student's understanding the higher the grade should be.

The system we have developed is based on learning objectives and was implemented in our Discrete Mathematics courses: a sequence of two courses primarily for Computer Science and Information Technology students.

Over the course of five semesters the courses have been taught by two professors, one with a computer science background and one with a mathematics background giving two different perspectives on the material and the experience of teaching using this rubric.

In this paper we describe the grading system, its benefits and challenges, and our experiences using it.

## RELATED WORK

Bloom's taxonomy is a hierarchy of learning [1]. The higher levels of the taxonomy typically indicate a deeper level of knowledge than the lower levels, and learners typically do not exhibit the higher levels without also exhibiting the lower levels. The six levels of Bloom's taxonomy are Knowledge (recollection), Comprehension, Application, Analysis, Synthesis, and Evaluation.

One revision of Bloom's Taxonomy gives the categories of learning from lowest to highest as Remember, Understand, and Apply, followed by Analyze, Evaluate, and Create [2]. These last three are given equal priority.

Johnson and Fuller ask whether Bloom's taxonomy is appropriate for computer science [3]. They raise the perspective that application is the primary aim of computing and as such, it is appropriate that there is a bias toward application in both instruction and assessment. Further, some of the people they interviewed suggested that the higher levels of Bloom's taxonomy are only reached during the final year of an undergraduate computing curriculum.

Lister and Leaney describe a process for incorporating Bloom's taxonomy into assessment for programming courses [4]. In their assessment, programming students earn the equivalent of a C for learning to read programs, a B for writing simple programs and program extensions, and an A for writing a larger programming project. Burgess applied Lister and Leaney's process in the setting of a small college [5].

## LEARNING OBJECTIVES

We developed learning objectives for each topic in the two-course Discrete Mathematics sequence (usually four learning objectives per topic). Each objective was rated based on the depth of understanding needed to pass it, and students were evaluated for each objective on a pass/fail basis.

Before developing the learning objectives we asked qualitatively what we expect an 'A' student, 'B' student, 'C' student, or 'D' student should be able to do. We came to the following conclusions. Examples of objectives follow.

A student who earns a D has not performed satisfactorily in the course. The student has not mastered the basic skills one would expect from a Discrete Mathematics student, but a D student has performed better than a student that failed the course. To earn a D, students should at least understand the problems that are being presented, even if solving the problems is beyond their skill set. A student should understand the language of Discrete Mathematics. We primarily tested this level of understanding through vocabulary comprehension. This corresponds with the learning category of Remember.

A student who earns a C has shown proficiency with the basic skills, but has not necessarily demonstrated complete mastery and has not been able to apply the skills to more difficult problems. They can read a proof and reproduce proofs that are very similar to what they have already studied, but do not have the skills to apply proof techniques in new contexts. To earn a C, a student should demonstrate proficiency with a core set of skills. This corresponds with the category of Understand.

A student who earns a B has demonstrated a mastery of the basic skills and proficiency with the advanced skills in the course, including completing proofs. This corresponds with the learning category of Apply.

A student who earns an A shows a mastery of all of the course material. The A student can formulate proofs on even difficult questions and apply their knowledge to new situations. Here we are including the last three learning categories Analyze, Evaluate and Create, whichever is most appropriate to the topic at hand.

With these ideas in mind about what a student should demonstrate to earn each grade, we developed learning objectives for each topic and each grade level.

For example, there are four learning objectives related to conversion between numeric bases:

1. I know these terms: base *b* expansion of *n*, binary expansion, hexadecimal expansion, and octal expansion.
2. I can convert a number from its decimal expansion to its binary or hexadecimal expansion. I can convert a number from its binary or hexadecimal expansion to its decimal expansion.
3. I can convert a number between any two bases. I can describe how to convert between different bases.
4. Given the description of a method of numeric representation (e.g. one's complement, two's complement, binary coded decimal, Cantor expansion), I can use it effectively.

The first learning objective (level D) deals with the vocabulary related to numeric bases, and does not require any depth of knowledge beyond memorization.

The second learning objective (level C) requires an understanding of basic algorithms to convert between base 10 and base 2 or 16 (the two most commonly used bases in computing). The second learning objective requires a deeper level of understanding because the students must understand the algorithm to use it.

The third learning objective (level B) requires students to apply their knowledge and extend the algorithm to any pair of arbitrary bases.

To earn credit for the fourth learning objective (level A), students must be able to apply any method of numeric representation that is described.

As with the numeric bases, other learning objectives are also organized into groups of four. Typically, the lowest level (level D) deals with the vocabulary of Discrete Mathematics, the next level (level C) deals with proficiency with basic skills, the third level (level B) deals with advanced skills or complete mastery of basic skills, and the fourth level (level A) deals with applying the skills to new topics or completing proofs about the topic.

Another example looks at the objectives for Rules of Inference. In order to meet the level D requirements students needed to know the following terms: valid argument form, rule of inference, fallacy, circular reasoning and premise. The level C objective required familiarity with the rules of inference listed in our text [7]. Correct identification of the rule of inference that was being used in an argument or being able to verify the argument was invalid was required for the level B objective. And the level A objective required students to apply rules of inference to draw relevant conclusions.

For each learning objective, sample problems from the textbook are provided to help guide students' learning. Students know that in order to pass that objective, they should be able to solve the given problems. Problems for quizzes and tests are largely (if not exclusively) drawn from the provided bank of questions with at most minor modifications. The learning objectives coupled with sample questions provide students with very clear expectations. The clear expectations enhance student accountability and allow students to always know where they stand in relation to the course expectations. Regular feedback and student accountability are good motivators to student learning [6].

## CLASS FORMAT

In this paper we discuss only an assessment method. The assessment does not alter the way that the material is presented. The course is taught in a traditional style using some combination of chalk and board, Power Point slides and problem solving with the students. Assessment is also given as in traditional courses with weekly quizzes and several tests throughout the semester, but the format and grading style of the assessment instruments is where our approach differs.

As our presentation of the material is no different from our previous classes there was no need to rewrite our class plans, thus minimizing the time required for adaptation. After creating the learning objectives for a section we might need small modifications to our notes, but these were similar to the modifications usually made when a course is taught over multiple years.

### EARNING CREDIT

There are multiple ways a student can prove their proficiency at a learning objective. We have employed three primary means: quizzes, tests, and challenge problems.

Firstly there are quizzes given at least weekly. These usually focus on a single topic and will contain D, C and occasionally B and A level objectives. Level D objectives were generally tested using match the word with the definition type questions. Weekly quizzes provide the students with rapid feedback on their progress with the material and help them to stay up to date with learning the basics so that they can follow the lectures more easily.

There are then regular tests on which the full range of levels are given and students need only complete those learning objectives they have yet to prove mastery of.

While quizzes and tests are the primary means of earning credit for learning objectives, students may also earn credit for learning objectives through challenge problems. These problems usually take the form of harder problems that are completed outside of class. Alternatively, a challenge problem could be a brief oral exam on a single topic where the student answers the questions face-to-face.

Challenge problems have been implemented in two different ways; firstly only for level A objectives with the student asking if certain problems from the text were appropriate. The second implementation allowed students to complete any level of objective through a challenge problem by answering an instructor assigned problem either from the text or created by the instructor. This second implementation allows for students to take as many attempts as they need to prove their proficiency in a topic.

One of us did allow unlimited attempts at any objective during the semester. This does become quite time consuming, particularly as you want to make sure that students are working individually on challenge problems so must assign different problems to different students and then grade them. This is manageable with a small class and truly embraces the philosophy that a student's final understanding is what matters, not the time it takes to get there. It is not recommended in larger classes and safeguards must be put in place so that serious procrastinators do not abuse the system and leave everything until the last minute.

With quizzes, tests, and challenge problems, students were given multiple opportunities to pass each learning objective. In the initial implementations a student's final grade was based solely on the number of learning objectives passed. Due to some of the challenges we faced this initial approach was adapted to meet the academic maturity of our students.

### GRADING CRITERIA

The learning objectives capture what the students should know to attain a particular grade in the course. Unfortunately, students do not proceed equally through all of the material in the course. One student may attain total mastery of one topic but struggle greatly with another topic. A different student could face the exact opposite situation. How do the learning objectives translate to course grades?

In the initial implementation of the course, students were required to pass all level D objectives to attain a D. To attain a C, students were required to pass all level D objectives and three-quarters of the level C objectives. To attain a B, students were required to pass all level C and D objectives and three-quarters of the level B objectives. To attain an A, students were required to pass all of the level B, C, and D objectives and three-quarters of the level A objectives.

Recognizing that a student who earns credit for a higher-level learning objective typically makes use of the skills from the corresponding lower-level learning objectives, a student who earned credit for a level B objective also was given credit for the corresponding level C objective. Similarly, a student who earned a level A objective automatically received credit for B and C as well.

This implementation had the intended effect of directing students to focus on the material that they did not already know well. If they had not yet passed the level C objective for one topic, they could not make up for it by doing well on another topic. They had to focus on the material that was most difficult for them.

An unintended consequence of the grading system was that the students experienced an undue amount of anxiety over the vocabulary quizzes and spent more time than necessary memorizing the terms. The vocabulary quizzes should have been easy for students since they simply covered the terms we used throughout our discussions in the course and the vocabulary portions were primarily matching or multiple choice. However, making the level D learning objectives absolute and non-negotiable led to students resorting to rote memorization. The students did well on the vocabulary, but for many students the time spent on rote memorization would have been better spent focusing on problem solving. (For other students, however, vocabulary memorization was an important first step in understanding the course material.)

In another implementation of the course, the final grade was based simply on a total count of learning objectives completed. Out of 100 learning objectives, students needed 25 for a D, 50 for a C, 75 for a B, and 90 for an A. In this iteration, students were able to make up for a deficiency in one area by a stronger performance in another. Another change was that students were no longer given credit for lower-level learning objectives simply by passing higher-level learning objectives.

This implementation solved the problem of undue emphasis on rote memorization. Students still studied the vocabulary (as they should have), but because two or three missed level D learning objectives would not cause them to fail the course, they did not focus on it to the extent that it displaced an effort on problem solving.

Removing the automatic passing of lower level objectives made sure that students had the broad depth of knowledge we desired and had not just learned to answer

type A questions without fully mastering type B and C which may focus on a slightly different area within the topic.

However, we were surprised by students who quit working once they had passed enough learning objectives to obtain their desired grade even if it was a C. In one case a student had an A- going into the final but chose not to take the final exam as they felt the work involved was not worth the reward.

In the latest implementation, 60% of the grade is based on progress with the learning objectives. The other 40% is split evenly between homework (graded for completeness and effort rather than correct answers) and a traditional cumulative final exam. We found these two measures to be beneficial to ensure that students keep up with the work throughout the semester and do not stop working near the end.

Knowing there is a cumulative final also discourages students from cramming before quizzes and tests in order to pass a specific objective with the view they will never have to worry about it again.

## BENEFITS

A grading system based on learning objectives offers several advantages. It is less arbitrary than giving partial credit, students directly see how their learning relates to their grades, students are graded based on their total progress instead of their rate of progress, and it is very clear to students what topics they need to work on.

This system allows students multiple opportunities to demonstrate their understanding and they are not penalized for not getting it the first time. Thus their grade is based only on their total knowledge and not on how fast they acquired it.

As learning objectives are graded only on a pass/fail basis, students will not receive partial credit. Points that are assigned for partial credit can often be arbitrary. For example, if a ten-point question is answered incorrectly, giving either 1 point or 5 points partial credit would both indicate a failure to correctly solve the problem. However, the difference between 1 and 5 points is nearly a half-letter grade on a 100 point test. It's certainly possible, and expected, to give students who make the same error the same amount of partial credit, but it is difficult (particularly on proofs) to determine whether a solution that includes a particular error is worth 0, 2, or 5 points. It is too easy for that decision to be arbitrary. The pass/fail nature of learning objectives removes that arbitrary decision. There is, of course, the question of where to draw the line for pass/fail, but because students have multiple chances to pass the learning objectives, initial failure does not necessarily harm a student's final grade.

Another benefit of learning objective based grading is that students are led to focus on the material they have not yet mastered and have very clear information about what will be on the tests and quizzes.

Weaker students can focus their energies on basic skills that they have not yet mastered without being distracted by material they are not yet ready to attempt anyway. It is even clear to them which types of problems a C student should be able to answer and a bad first test is not seen as a reason to give up. Stronger students can move on to the advanced material more quickly.

This ability to meet the needs of both the strong and weak students in a class is a major strength of this system. It helps to reduce the boredom that the strongest students can experience and provides them with more challenge than tests aimed at the average student often do. For the weaker student the clear objectives allow them to work at a level they feel comfortable with without the constant feeling that the class is moving too fast for them and they will never catch up.

Instructors are required to focus deeply on what the objectives for the course are. Taking the time to classify problems in the text by the depth of understanding required is challenging, but helps the instructor to solidify the goals for the course and the students.

Most importantly, grades are more directly linked to depth of understanding. We know that our A students have demonstrated an appropriate depth of knowledge to deserve their grades not simply outperformed their fellow students. As our class size is generally around a dozen this helps with the disparity in students' ability from semester to semester, and as both student and instructor are very clear on what is needed to achieve a specific grade there are no pleas for higher grades come semester end.

## CHALLENGES

The initial creation of learning objectives is time consuming and if the professor desires to associate learning objectives with sections in a textbook, they may need to be adjusted when new editions of the text book are published or if a different text is used.

Partial credit can be an important piece of feedback for students, indicating how well they understand the material. Because each problem in our grading system is graded on a pass/fail basis there is no partial credit. Thus it is more important to give meaningful feedback on graded assignments and to encourage students to review it so they know where further work is needed.

In order to include all of the learning objectives on a test, the tests can be quite lengthy. Students must be reminded to focus on the learning objectives they have yet to meet and that they are not expected to answer every question on the test. The top students will have already passed most of the level C and many level B objectives on the weekly quizzes. The weaker students, who are often attempting the level C and level B objectives again, will probably not have time to complete all of the questions related to the level A objectives, but most of them would not be able to answer those questions anyway. (However, the weaker students who are particularly motivated can still earn credit for the higher-level objectives through challenge problems after the test.)

Students and instructors also need to keep accurate records of which learning objectives have been met. If a student forgets to bring this information to a test or has not updated their sheet they can waste valuable time redoing objectives they have already met. On the other hand, the time is only wasted in the sense that it does not help their grade and may prevent them from getting to other questions on the test. Redoing learning objectives that have already been passed is still valuable practice.

As there is no penalty for failed attempts, other motivation is needed to ensure that students keep up with the material. Some such approaches are discussed above.

We were also surprised at how challenging some students found the vocabulary sections on tests. As these were tested with either multiple choice or by matching the appropriate word from a given list to the correct definition we expected that students would find this quite straightforward. However, there were cases where students were unable to meet this objective but were able to obtain the C and B level objectives for the same topic. This does perhaps demonstrate the challenge of learning to speak mathematics. Understanding the difference between "at least one" and "at most one" within a definition can be harder than visually recognizing a one-to-one or onto function.

There can also be a tendency for some students to leave everything until the last minute if you allow them too many opportunities to pass an objective. This results in cramming which is not beneficial to long term learning. Again, some of the modifications discussed above help to handle this issue.

As with any learning approach which is unfamiliar to students some resistance is to be expected and clear descriptions of the process are essential for student comfort.

## STUDENT COMMENTS

We provide a sampling of comments from student evaluations in various semesters of the course. We have edited the comments only for grammar and included both the very positive ones and those that were less enthusiastic. They clearly show that the student perspective to the rubric is very similar to ours. Of the comments relating directly to assessment, positive comments outweighed negative comments by almost two-to-one, and the majority of negative comments complained that the grading system made it too hard to earn an A.

"The course allowed me time to learn and earn the objectives. I liked being able to learn at my own pace and the many opportunities to get the objective."

"Instructor was fine, just needs another semester with the grading system to get it to its full potential."

"Less learning objectives"

"Learning objectives are awesome."

"The unconventional grading system definitely helped in this course."

"I think the Learning Objective grading system is weird. If you get one part of a section wrong, most of the time, the entire section will be marked wrong regardless of if everything else was correct."

"Good grading system to allow the student a second chance to learn the material."

"Grading system – too much of opportunity for failure or grade of a 'D'. The grading practice would be better for a graduate level class."

"The grading system is a fair system."

"The grading scale could be improved to make it easier to obtain an A."

"Students can easily determine their grade and know what they need to work with in order to get a good grade with the experimental grading scale."

"Learning Objective System is very good; second time taking course, first time using the learning objective system. The learning objective system is much better."

## CONCLUSIONS

Learning objectives are a good way to implement Bloom's Taxonomy, but some structure must be maintained to avoid students who abuse the system and those who are not ready to take control over their own learning.

By semester end most students find this a fair system of grading and it is extremely popular with those students who work hard, but need time to develop a deep understanding of material.

## ACKNOWLEDGMENT

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**AUTHOR INFORMATION**

**Timothy Highley,** Assistant Professor of Computer Science, La Salle University, highley@lasalle.edu.

**Anne E. Edlin,** Assistant Professor of Mathematics, La Salle University, edlin@lasalle.edu