Work in Progress: *i*Comb Project - a mathematical widget for teaching and learning combinatorics through exercises

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***Abstract* - This paper presents *i*Comb, a widget for teaching and learning combinatorics. The project was inspired by the idea of bringing up mathematical domains to the changing trend named as Web 2.0 and encouraging the development of a family of "math widgets" to improve the mathematical education scenario. The paper's contribution are: (i) a web-based application that is tailored to learn and teach combinatorics, an important issue for K12 and CSTE students; (ii) an interactive widget that can enrich web sites and e-communities; (iii) an e-LM (e-Learning Module) with automatic assessment resources.**

ICOMB

The *i*Comb Project provides a web "widget" that can help students to learn how to solve combinatoric exercises and understand their related concepts.

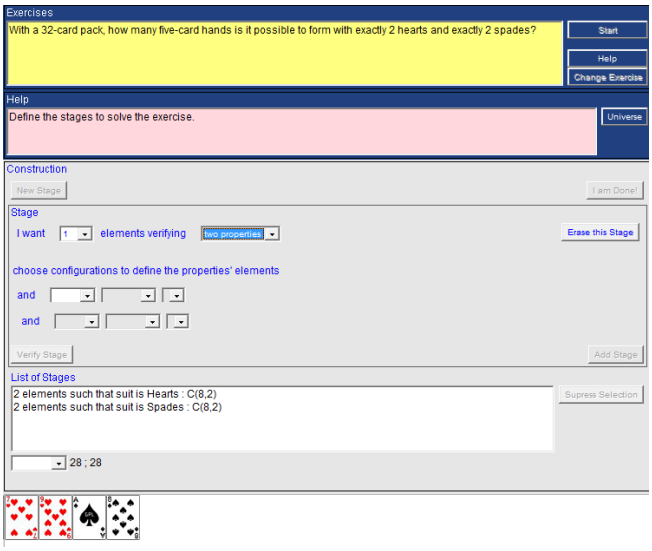

Figure 1: *i*Comb Widget

A widget is a mini-application that can be easily added to Web sites by using the copy-and-paste pattern. Therefore, integrating an *i*Comb widget into a blog or even in a complex LMS (Learning Management System) is as simple as copying a piece of HTML. It can also be used as an e-LM (e-Learning Module) under SAW, a web-based LMS, designed to increase interactions with learners [1]. The figure 1 present the *i*Comb widget. The figure 2 show the *i*Comb web page where the widget code can be copied from.

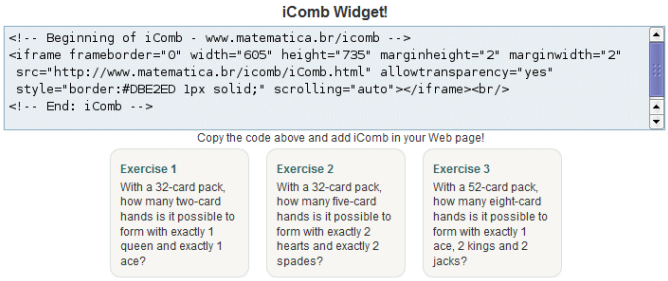

Figure 2: *i*Comb Project web page

The project was inspired by the idea of bringing up mathematical domains to the changing trend named as Web 2.0 and encouraging the development of a family of "math widgets" to improve the mathematical education scenario. It is observed that the interactive widgets for mathematical education are rare. Besides, there is a strong demand for collaborative tools that can be easily shared in social communities.

Combinatorics is a subject of K12 and of some undergraduate courses. Despite its importance to motivate students to CSTE and Math related careers, the literature claims that it is a difficult issue to teach and learn [2].

The main *i*Comb goal is to improve the teaching and the learning of combinatorics by offering a modern approach that brings such domain to the World Wide Web. Moreover, *i*Comb aims at providing a powerful tool to help teachers

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to teach and evaluate the student's development in combinatorics. Nevertheless, when *i*Comb is integrated to a repository system, its content can be shared, providing collaboration among teachers/instructors that are interested on combinatorics.

The *i*Comb solving engine uses the "constructive method" defined by the "Combien?" group from Universitè of Paris IV - LIP6 [2, 3]. Using this method, students are able to model the problems and their solutions and become conscious of the underlying mathematical elements concealed behind their everyday language. *i*Comb extends the "Combien?" project in four different ways: (1) it brings to the Web the "Combien?" model; (2) it explores the concept of dynamic repository of exercises; (3) it defines a plugin architecture that allows a multi domains paradigm; and (4) it explores a new image model that allows students really see the results of their reasoning during the resolution of a particular exercise.

The *i*Comb system is freely distributed and its first prototype is available under http://www.matematica.br/icomb.

AUTOMATIC EVALUATION

The current version of *i*Comb works with problems that can be decomposed in finite number of stages similar to the theorem 1, presented bellow. Any considered problem is to find "how many elements exists"? For example, if the universe is a 32-card pack, an *i*Combproblem can be "Using a 32-card pack, how many four-card hands is possible to form with 1 aces and 3 queens?"

To solve an *i*Comb problem, the student should define a set of *stages*. A *stage* is applied to a set and has an a number called *gauge* as an extra parameter. Being D a set of elements, a *stage* of D with *gauge* g is represented by $[D,g]$ and defined as $[D,g] = \{d \subset D : |d| = g\}$. D is also called *domain* of a stage and can be referred by $\mathrm{dom}([D,g]) = D$ while the *gauge* of a *stage* can be expressed by $\mathrm{g}([D,g]) = g$.

Let $D_1 = \{\stackrel{A}{\clubsuit}, \stackrel{A}{\heartsuit}, \stackrel{A}{\spadesuit}, \stackrel{A}{\diamondsuit}\}$ and $D_2 = \{\stackrel{Q}{\clubsuit}, \stackrel{Q}{\heartsuit}, \stackrel{Q}{\spadesuit}, \stackrel{Q}{\diamondsuit}\}$. The answer of the above problem is the cardinality of $[D_1,1] \otimes [D_2,3]$ where $\otimes$ represents the cartesian product and:

$$[D_1,1] = \{\{\stackrel{A}{\clubsuit}\}, \{\stackrel{A}{\heartsuit}\}, \{\stackrel{A}{\spadesuit}\}, \{\stackrel{A}{\diamondsuit}\}\}$$

$$[D_2,3] = \{\{\stackrel{Q}{\heartsuit}, \stackrel{Q}{\spadesuit}, \stackrel{Q}{\diamondsuit}\}, \{\stackrel{Q}{\clubsuit}, \stackrel{Q}{\spadesuit}, \stackrel{Q}{\diamondsuit}\}, \{\stackrel{Q}{\clubsuit}, \stackrel{Q}{\heartsuit}, \stackrel{Q}{\diamondsuit}\}, \{\stackrel{Q}{\clubsuit}, \stackrel{Q}{\heartsuit}, \stackrel{Q}{\spadesuit}\}\}$$

A *solution* to a *i*Comb problem is a set of *stages* $\{S_1, S_2, \ldots, S_k\}$ where:

$S_1 \otimes S_2 \otimes \ldots \otimes S_k$ is the set of all configurations asked in the words.

$S_1, S_2, \ldots, S_k$ are pairwise disjoint.

Theorem 1 *If $S = \{S_1, S_2, \ldots, S_k\}$ is a solution for a iComb problem, a set of stages $Z = \{Z_1, Z_2, \ldots, Z_m\}$ will be also a solution if:*

- $\{x \in S : 0 < \mathrm{g}(x) < |\mathrm{dom}(x)|\} = \{x \in Z : 0 < \mathrm{g}(x) < |\mathrm{dom}(x)|\}$
- $\bigcup_{\substack{x \in S \\ \mathrm{g}(x)=0}} \mathrm{dom}(x) = \bigcup_{\substack{x \in Z \\ \mathrm{g}(x)=0}} \mathrm{dom}(x)$
- $\bigcup_{\substack{x \in S \\ \mathrm{g}(x)=|\mathrm{dom}(x)|}} \mathrm{dom}(x) = \bigcup_{\substack{x \in Z \\ \mathrm{g}(x)=|\mathrm{dom}(x)|}} \mathrm{dom}(x)$

The theorem 1 is very important to *i*Comb, since it is the basis to automatic evaluation algorithms of *i*Comb. This algorithm is very fast and can detect any wrong solution or any right solution. A right solution could use any number of stages, since it produces the correct final set.

CONCLUSIONS

*i*Comb pursues the goal to improve the teaching and the learning of combinatorics by offering a modern approach in order to expose such a domain to the World Wide Web. Moreover, *i*Comb aims at providing a powerful tool to help teachers to teach and evaluate the student's development in combinatorics. Nevertheless, when *i*Comb is integrated to a repository system, its content can be shared, providing collaboration among teachers/instructors that are interested on combinatorics.

The *i*Comb is currently in use as an e-Learning Module of SAW [1] and was already tested in two classes.

Acknowledgment

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