INTERACTIVE FOURIER CONCEPTS: COMPUTING TOOLS FOR BUILDING INTUITION

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Abstract *– Fourier concepts for describing the frequency content of signals permeate contemporary engineering and science. The introduction of Fourier series and transforms, as well as discretization and sampling topics, is seen by many educators to be essential for providing the next generation of engineers with a foundation for better understanding the systems they deal with, not to mention the various knobs and functionality provided by contemporary instrumentation. To assist the pedagogical effort, the authors have developed a set of GUI-based applications which allow students to interact with the frequency content of simple as well as quite complex signals. The applications have been constructed using MATLAB, and a companion website hosts them. They are "free-for-the-taking," and have been well received in several local forums to date.*

Index Terms – DFT/FFT, Fourier, GUI applications, MATLAB.

INTRODUCTION

Once the lone bastion of signals and systems coursework in electrical engineering, continuous and discrete Fourier frequency domain concepts are showing up in diverse course offerings covering the spectrum of undergraduate engineering and science curricula. While also once relegated to senior and graduate level courses, digital signal processing concepts have begun to find a foothold in the freshman and sophomore years, even, believe it or not, as topics designed to engage and capture the interests of our multimedia-crazed youth [1]-[2]. This advancement in topics has been attributed to the power and ubiquitous nature of computing hardware. A quick scan of contemporary marketing literature for even the lower-end bench-top oscilloscope will reveal a wealth of signal processing capability at the fingertips of the pubescent engineer [3]-[4]. Even closer to home, very few personal computers are now sold without the inherent ability to sample signals, certainly audio if not also video, and complete very powerful analyses at the click of a mouse.

In order to correctly apply these tools, a better than cursory familiarity with the underlying concepts of frequency content and sampling is demanded. To assist the pedagogical effort, the authors have developed a set of GUI-based applications which allow students to interact with the frequency content of simple as well as quite complex signals. Building on related efforts described in the literature [5]-[9], the authors have appealed to various types of learning styles, incorporating visual as well as auditory domains for frequency analysis. The applications have been constructed using MATLAB, utilizing standard I/O capabilities to enhance portability and applicability. A companion website hosts these applications:

http://wwweng.uwyo.edu/electrical/dsp_audio

The applications work with MATLAB 6.0 and higher and are free-for-the-taking. The independent, exploratory nature of the applications has proven to be a useful tool for enhancing learner intuition and subsequent mastery of topics.

THE APPLICATIONS

The applications are summarized briefly in Table I. The order of the applications in this table reflects the classical pedagogical progress through the topic areas of continuous and discrete Fourier studies.

TABLE I
THE GUI APPLICATION SET

Application Name	Description
Inner Product	Computation and plotting of orthogonal functions and products
Harmonic Synthesizer	Computation, plotting and audio playback of user prescribed periodic signals
Sampling and Aliasing	Computation and overlay of sampled sinusoids with their aliasing counterparts
Aliasing Motion	Computation and video replay of "spoked wheel" video framing alias
Basic DFT/FFT	Basic computation of the Discrete Fourier Transform: limited parameter adjustment interface for the effects of sampling rate, number of samples, zero padding, and function type
Advanced DFT/FFT	Advanced computation of the Discrete Fourier Transform: full parameter adjustment interface to the effects of sampling rate, number of samples, windowing functions, and function type

Inner Product

The concept of vector orthogonality in a finite dimensional, Cartesian space is typically comfortable material for freshman and sophomore students with early Trigonometry and Calculus backgrounds. Making the leap to orthogonality of functions and infinite dimensional function spaces leaves some of this group behind. The Inner Product application is designed to provide interactive manipulation of the Fourier function basis (sinusoids of various frequencies and phases).

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Appealing to the graphical interpretation of integration, this GUI tool helps to instill the critical concepts of harmonic number, phase, and fundamental period. The interface and a demonstration case are shown in Figure 1.

GUI Design and Evaluation Comments: The graphical display of the intermediate integration step supplements learning through visual reinforcement of an otherwise abstract calculus concept. Providing various test functions and flexibility in selection of frequency promotes intuitive learning where what-if and immediate feedback are supported. Student comments include recognition that "zero now comes in a lot of shapes and sizes." Some students even admit that waveforms begin to suggest that mathematics include some aesthetically pleasing elements.

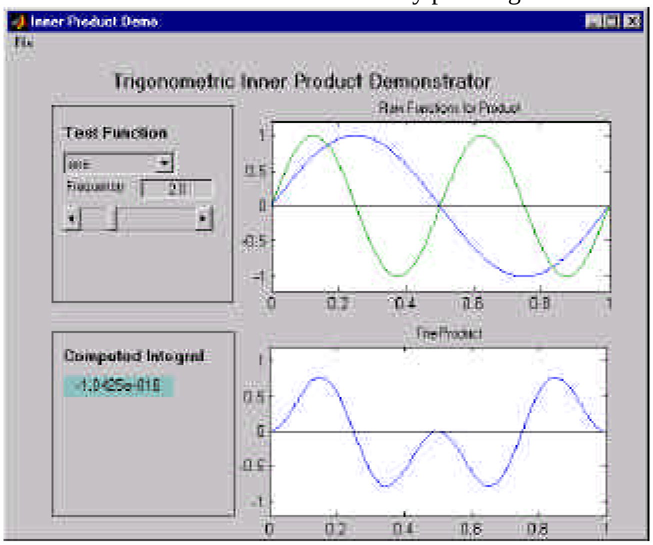

FIGURE 1
THE INNER PRODUCT APPLICATION.

Harmonic Synthesizer

The framework of "sums of harmonically related sinusoids" can be brought to life via both graphical and auditory means. The Harmonic Synthesizer application allows the user to prescribe the harmonic mix of a signal, vary the fundamental frequency and time duration, then "play" the signal back through conventional personal computer audio systems. The interface and a demonstration case are shown in Figure 2.

GUI Design and Evaluation Comments: The graphical display of the partial sum is the classical strength of this synthesizer, however it does not stand alone. By adding the audio playback feature, another sense is integrated into the task. Students prompt one another to "listen to what I've created, and guess how I did it, don't peek!" The format of the Harmonic Recipe Selector is reasonably open-ended, which invites students to generate examples far from the lecture and text material. Finally, because all computer audio systems are not identically capable, students are led to discover the frequency effects of the measurement and generation equipment.

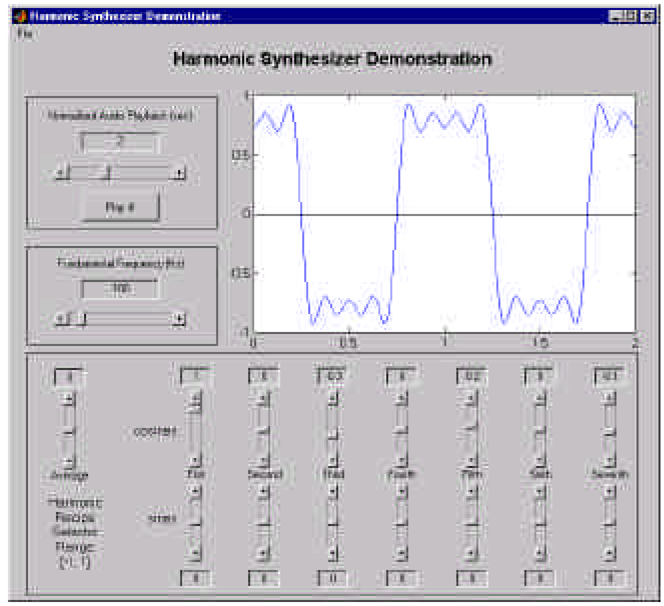

FIGURE 2
THE HARMONIC SYNTHESIS APPLICATION.

Sampling and Aliasing

Moving into time sampling considerations is a critical step, and in fact, many have argued that we must do this early (the underlying concepts are algebraic in nature, and the consequences of not being familiar with the phenomenon of aliasing can be serious). The Sampling and Aliasing application allows the user to prescribe the sample rate for a single cycle of a pure sinusoid then to view as many aliases as one finds useful. The interface and a demonstration case are shown in Figure 3.

GUI Design and Evaluation Comments: The mainstay of graphical/visual enforcement is obvious here, and the limitless nature of the aliasing effect is recognized in student comments ("It's now obvious that there's a bunch out there.")

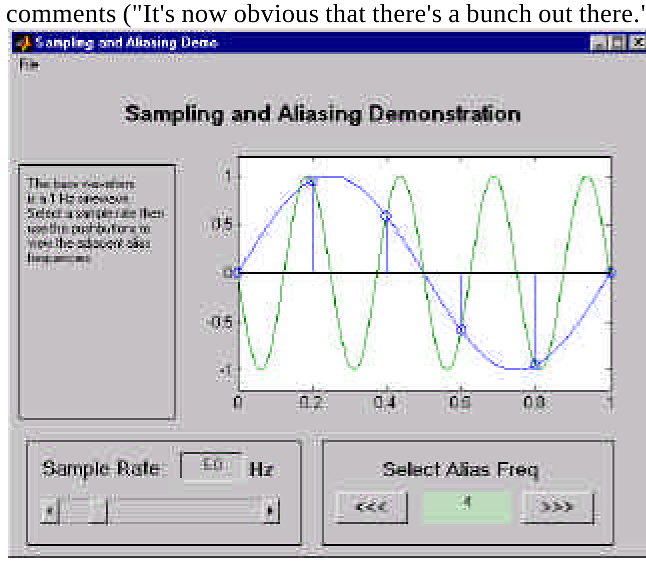

FIGURE 3
THE SAMPLING AND ALIASING APPLICATION.

Aliasing Motion

For many of us, our first view of the effects of aliasing came while watching the spokes of a wagon wheel appear to rotate backwards as our hero/villain undertook yet another dramatic chase across the western landscape projected on the silver screen or television tube. This effect also assists in the investigation of "positive and negative" frequency concepts. The Aliasing Motion application allows the user to prescribe the rotational rate of the "wheel" as well as the "frame (sampling) rate." This combination provides for a very broad set of aliasing conditions. Playback on contemporary computing platforms has proven to be essentially real-time. The interface and a demonstration case are shown in Figure 4.

<u>GUI Design and Evaluation Comments:</u> As in the previous GUIs, graphical/visual enforcement is the primary vehicle, however this GUI takes it one more step into the time evolution of the aliasing phenomenon. This GUI plays out in effective real-time, with enough student control to allow for discovery of the parameter mix which leads to "fooling the eyes." The analytical periodicity of aliasing is drawn out here through visual enforcement of the continuum of rates and directions of apparent rotation. Students really enjoy this demonstration, and they report an interest in examining other techniques for fooling their cohorts.

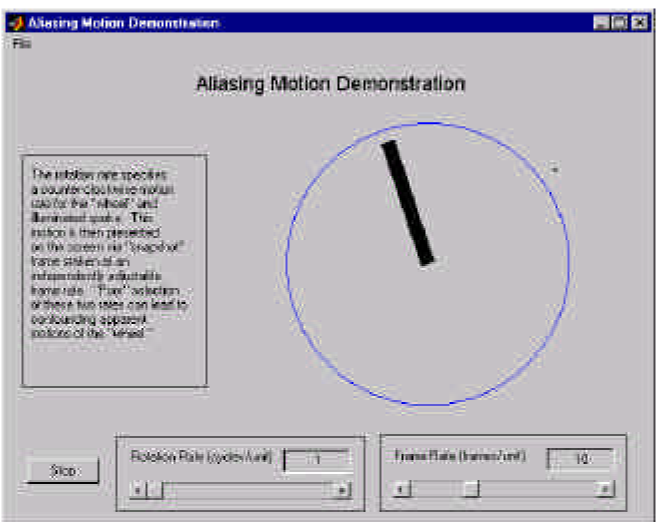

FIGURE 4
THE ALIASING MOTION APPLICATION.

Basic DFT/FFT

The Discrete Fourier Transform lies at the heart of many contemporary analyses (and in the guts of many contemporary instruments). Those who are partially "in the know" are at least familiar with the acronym FFT (Fast Fourier Transform), however few have a good working knowledge of the parametric dependencies of these techniques. The Basic DFT/FFT application allows the user to select a fundamental signal type, the sample rate and duration of sampling, any zero padding to be inserted before computing the FFT, then several formats for display of the frequency domain results. Through the variety provided, concepts as diverse as aliasing, frequency resolution and sidelobe leakage can be investigated. The interface and a demonstration case are shown in Figure 5.

<u>GUI Design and Evaluation Comments:</u> The number of serial processes involved in arriving at the final analytical result anaesthetizes students frequently. Through careful, coordinated display of the serial processes and their visual outcomes, the global effects of the smallest steps can be viewed and compared. This GUI "brings it all together" by allowing all parameter variations. Students are initially surprised that they can enter parameters which result in aliasing, for example, where the global result is invalid. Recognizing that they are in fact twiddling knobs similar to those on the face of the bench-top instruments promotes an awareness of how they can be responsible for allowing the instrument to lie.

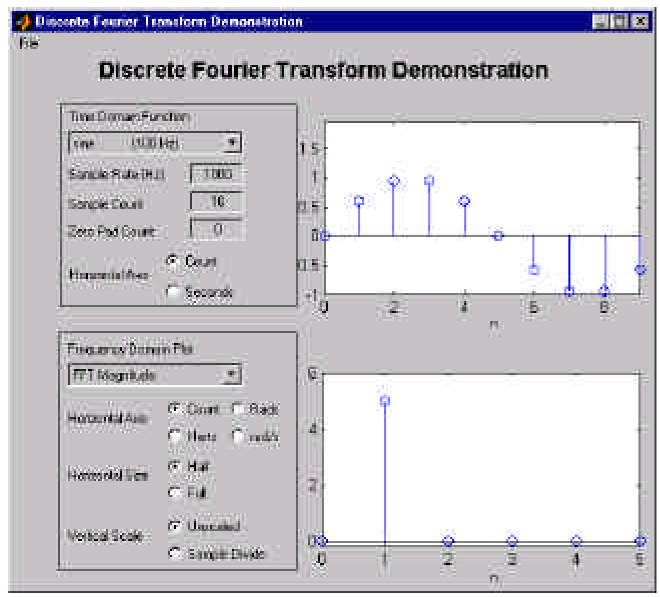

FIGURE 5
THE BASIC DFT/FFT APPLICATION.

Advanced DFT/FFT

Following on the Basic DFT/FFT, the final concept of data windowing is examined with the help of the Advanced DFT/FFT application. The window choice is becoming a very common knob or soft selector on a host of analytical instruments. This application provides the user with an interactive laboratory for viewing the effects of the choice of several common windowing techniques. The interface and a demonstration case are shown in Figure 6.

<u>GUI Design and Evaluation Comments:</u> This GUI compounds the number of serial processes. By advancing slowly to this final tool, the ultimate assemblage of bells and whistles is no longer as intimidating as it would have been.

Exercises using common test signals can be used to enforce student confidence with similar instrument front panels in the future ("I think I know what to expect for this signal, so I can try it now"). Addition of noise components to the pure signals will be included in future revisions, thanks to student comments on the effects viewed on actual instruments.

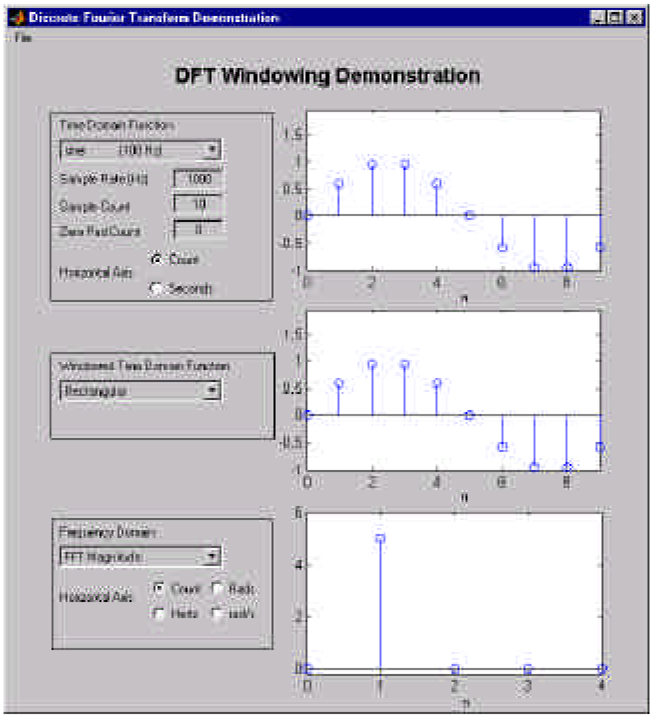

FIGURE 6
THE ADVANCE DFT/FFT APPLICATION.

SOME TECHNICAL ISSUES

The MATLAB GUI development for these applications was completed with the MATLAB Version 6 GUIDE (GUI DEVelopment environment). The GUIs have been tested on Windows NT, 2000 and Linux X/Windows environments, including standard sound card installations. Only basic MATLAB functions are used (no toolboxes required).

SUMMARY

The GUI applications described above have been very useful in the development of student intuition and confidence with continuous and discrete Fourier frequency domain topics. These fundamentals are essential for intelligent and accurate use of contemporary instrumentation, by all disciplines, and the GUI/multimedia mix is designed to match the expectations of our current generation of students as well as the computational hardware and software available in our laboratories and on our laptops.

Student feedback and evaluation to date consists of written end-of-class evaluations as well as lab notebook summary comments. Comparison of student performance with these supplementary GUIs versus without has not yet been completed.

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